

# Linear Inflation from Running Kinetic Term in Supergravity

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## Abstract

We propose a new class of inflation models in which the coefficient of the inflaton kinetic term rapidly changes with energy scale. This naturally occurs especially if the inflaton moves over a long distance during inflation as in the case of large-scale inflation. The peculiar behavior of the kinetic term opens up a new way to construct an inflation model. As a concrete example we construct a linear inflation model in supergravity. It is straightforward to build a chaotic inflation model with a fractional power along the same line. Interestingly, the potential takes a different form after inflation because of the running kinetic term.

The inflation has been strongly motivated by the observation [1], while it is a non-trivial task to construct a successful inflation model. A successful inflaton model must explain several features of the density perturbation, but properties of the inflaton are not well understood. It is often assumed that, in the slow-roll inflation paradigm, the inflaton is a weakly coupled field, and therefore the kinetic term is simply set to be the canonical form during inflation. This seems justified because the typical energy scale of inflation is given by the Hubble parameter, which remains almost constant during inflation. However, there is another important energy scale, namely, the inflaton field. Even in the slow-roll inflation, the motion of the inflaton is not negligible and it may travel a long distance during the whole period of inflation. In particular, in the case of large-scale inflation such as a chaotic inflation model [2], the inflaton typically moves over a Planck scale or even larger within the last 60 e-foldings [3]. Then, it seems quite generic that the precise form of the kinetic term changes during the course of inflation. In some cases, the change could be so rapid, that it significantly affects the inflaton dynamics. In this letter, we construct a model in which the coefficient of the kinetic term grows rapidly with the inflaton field value, but in a controlled way. By doing so, we construct a linear term inflation model in the supergravity framework (see Ref. [4] for the quadratic model). The realization of the linear term inflation model in the string theory was given in Ref. [5]. We also show that a chaotic inflation model with a fractional power can be straightforwardly constructed along the same line.

Before going to a realistic inflation model, let us give our basic idea. Suppose that the inflaton field  $\phi$  has the following kinetic term,

$$\mathcal{L}_K = \frac{1}{2}f(\phi)\partial^\mu\phi\partial_\mu\phi, \quad (1)$$

and that the inflaton field is canonically normalized at the potential minimum:

$$f(\phi_{\min}) = 1. \quad (2)$$

However, this does not necessarily mean that  $f(\phi)$  remains close to 1 during inflation, especially if the inflaton moves over some high scale, e.g., the GUT or Planck scale. Suppose that the behavior of  $f(\phi)$  can be approximated by  $f(\phi) \approx n^2\phi^{2n-2}$  with an integer  $n$  over a certain range of  $\phi$ . Then, when expressed in terms of the canonically normalized inflaton field,  $\chi \equiv \phi^n$ , the scalar potential  $V(\phi)$  is modified to be

$$V(\phi) \rightarrow V(\chi^{1/n}). \quad (3)$$

For instance, if  $n = 2$ , the quadratic potential,  $V(\phi) \propto \phi^2$ , becomes a linear term  $V(\chi) \propto \chi$ . Therefore, such a strong dependence of the kinetic term on the inflaton field changes the inflation dynamics significantly. In particular, the large coefficient of the kinetic term is advantageous for inflation to occur, since the effective potential becomes flatter.

Now let us construct a linear term inflation model in supergravity. In this inflation model, the inflaton field has a scalar potential linearly proportional to the inflaton,  $V(\phi) \propto \phi$ . For the inflation to last for the 60 e-foldings, the inflaton field must take a value greater than the Planck scale, which is difficult to implement in supergravity because of the exponential prefactor in the scalar potential. Therefore we need to introduce some sort of shift symmetry, which suppresses the exponential growth of the potential.

We introduce a chiral superfield,  $\phi$ , and require that the Kähler potential for  $\phi$  is invariant under the following transformation;

$$\phi^2 \rightarrow \phi^2 + \alpha \quad \text{for} \quad \alpha \in \mathbf{R} \quad \text{and} \quad \phi \neq 0, \quad (4)$$

which means that a composite field  $\chi \sim \phi^2$  transforms under a Nambu-Goldstone like shift symmetry. This is equivalent to imposing a hyperbolic rotation symmetry (or equivalently  $\text{SO}(1,1)$ ) on  $(\phi_R, \phi_I)$ , where  $\phi_R$  and  $\phi_I$  are the real and imaginary components,  $\phi = (\phi_R + i\phi_I)/\sqrt{2}$ .

The Kähler potential must be a function of  $(\phi^2 - \phi^{\dagger 2})$ :

$$K = ic(\phi^2 - \phi^{\dagger 2}) - \frac{1}{4}(\phi^2 - \phi^{\dagger 2})^2 + \dots, \quad (5)$$

where  $c$  is a real parameter of  $O(1)$  and the Planck unit is adopted. Note that the  $|\phi|^2$  term, which usually generates the kinetic term for  $\phi$ , is forbidden by the symmetry. Instead, the kinetic term arises from the second term, and the coefficient of the kinetic term will be proportional to  $|\phi|^2$ . Note that the lowest component of the Kähler potential vanishes for either  $\phi_R = 0$  or  $\phi_I = 0$ . This feature is essential for constructing a chaotic inflation model in supergravity.

Let us add a symmetry breaking term,  $\Delta K = \kappa|\phi|^2$ , to cure the singular behavior of the Kähler metric at the origin. Here  $\kappa \ll 1$  is a real numerical coefficient, and the smallness is natural in the 't Hooft's sense [6]. There could be other symmetry breaking terms, but, throughout this letter we assume that those symmetry breaking terms are *soft* in a sense that the shift symmetry remains a good symmetry at large enough  $\phi$ . The kinetic term of

the scalar field then becomes

$$\mathcal{L}_K = (\kappa + 2|\phi|^2 + \dots) \partial^\mu \phi^\dagger \partial_\mu \phi, \quad (6)$$

where the higher-order terms expressed by the dots contain terms proportional to  $(\phi^2 - \phi^{\dagger 2})$ . Let us drop the higher-order terms for the moment. As demonstrated later, the higher-order terms do not change the form of the kinetic term. For a large field value  $|\phi| \gg \sqrt{\kappa}$ , the coefficient of the kinetic term grows with the field value, which makes the potential flatter. The canonically normalized field is  $\chi = \phi^2/\sqrt{2}$ , as expected. In a sense,  $\chi$  is a more suitable dynamical variable to describe the system satisfying the shift symmetry (4). On the other hand, for a small field value of  $|\phi| \ll \sqrt{\kappa}$ , the canonically normalized field is  $\sqrt{\kappa}\phi$ . Thus,  $\phi^2$  and  $\phi$  are the dynamical variables for high and low scales, respectively.

We can interpret the above phenomenon in the following way. If we go to high energy scales, namely the large field value  $\phi$ , the self-interaction in the Kähler potential becomes strong, and the scalar field forms a bound state  $\phi^2$ . On the other hand, as  $\phi$  becomes small, the self-coupling becomes smaller and the symmetry-breaking term becomes more relevant. Thus  $\phi^2$  breaks up and  $\phi$  becomes the suitable variable. Such a phenomenon of forming a bound state seems quite generic if one considers a large-scale inflation in which the inflaton takes a very large field value during inflation. Because of the strong self-interactions, the inflaton kinetic term runs with scales, and the inflaton dynamics is significantly changed. The novelty here is the existence of the shift symmetry, without which we cannot control the effect of the higher order terms on the inflationary dynamics. We will come back to this point later.

In order to construct a realistic inflation model, we consider the following Kähler and super-potentials <sup>1</sup>

$$K = \kappa|\phi|^2 + ic(\phi^2 - \phi^{\dagger 2}) - \frac{1}{4}(\phi^2 - \phi^{\dagger 2})^2 + |X|^2, \quad (7)$$

$$W = mX\phi, \quad (8)$$

where both  $\kappa$  and  $m$  break the symmetry, and so we assume  $\kappa \ll 1$  and  $m \ll 1$ .<sup>2</sup> These

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<sup>1</sup> One can add an additional breaking term in the superpotential which induces a periodic potential for  $\varphi$  [5]. The interesting feature may be found in the non-Gaussianity in this case [7].

<sup>2</sup> The breaking of the shift symmetry in the superpotential could produce radiative corrections to the Kähler potential. In particular,  $\kappa = O(m^2)$  is induced [4]. Here we consider a more general case that  $\kappa$  and  $m$  are not related to each other.

small parameters are naturally understood in 't Hooft's sense. The superpotential produces the inflaton potential. We assume that  $X$  and  $\phi$  have  $U(1)_R$  charges 2 and 0, respectively. We assign a  $Z_2$  symmetry under which both  $X$  and  $\phi$  flip the sign.

The Lagrangian is given by

$$\mathcal{L} = (\kappa + 2|\phi|^2)\partial_\mu\phi^\dagger\partial^\mu\phi + \partial_\mu X^\dagger\partial^\mu X - V \quad (9)$$

with

$$V = e^K \left( |D_X W|^2 K^{X\bar{X}} + |D_\phi W|^2 K^{\phi\bar{\phi}} - 3|W|^2 \right). \quad (10)$$

The scalar potential looks complicated, but it can be reduced to a simple form during inflation. One can show that, during inflation,  $X$  acquires a mass of the order of the Hubble scale and is stabilized at the origin, if  $|c| \gtrsim 1$ .<sup>3</sup> Then the scalar potential becomes

$$V \approx \frac{1}{2} e^{\frac{\kappa}{2}(\phi_R^2 + \phi_I^2) - 2c\phi_R\phi_I + \phi_R^2\phi_I^2} m^2 (\phi_R^2 + \phi_I^2). \quad (11)$$

The flat direction is given by  $\phi_R\phi_I = \text{constant}$  because of the symmetry (4). Therefore if  $\phi_R$  has a very large value,  $\phi_I$  is stabilized at a point where the Kähler potential is minimal. For  $\phi_R > 1$  and  $\kappa \ll 1$ ,  $\phi_I$  is stabilized at

$$\phi_I \approx \frac{c}{\phi_R}. \quad (12)$$

Here and in what follows we focus on the case of  $\phi_R > 0$  and  $\phi_I > 0$  without loss of generality. The scalar potential is then reduced to the following form:

$$V \approx \frac{1}{2} e^{\frac{\kappa}{2}\left(\phi_R^2 + \frac{c^2}{\phi_R^2}\right) - c^2} m^2 \left( \phi_R^2 + \frac{c^2}{\phi_R^2} \right), \quad (13)$$

for  $|\phi_R| > 1$ . Since we explicitly breaks the shift symmetry (4) by the  $\kappa$  term, there appears a non-vanishing exponential prefactor. However, for  $|\phi_R| < 1/\sqrt{\kappa}$ , the exponential prefactor is of  $O(1)$ , and therefore can be dropped.<sup>4</sup> The Lagrangian for the inflaton  $\phi_R$  is summarized by

$$\mathcal{L} \approx \frac{1}{2} \phi_R^2 (\partial\phi_R)^2 - \frac{1}{2} m^2 \phi_R^2, \quad (14)$$

<sup>3</sup> A self-coupling  $\sim |X|^4$  in the Kähler potential produces a Hubble-induced mass term for  $X$  about the origin.

<sup>4</sup> Actually, the inflaton does slow-roll if the exponential prefactor gives a main contribution to the tilt of the potential.

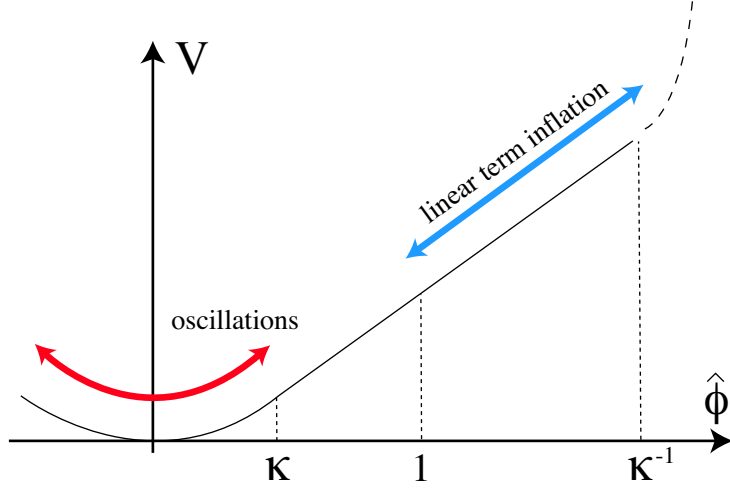


FIG. 1: The schematic plot of the scalar potential in terms of a canonically normalized field,  $\hat{\phi}$ . The potential is quadratic around the origin, and linear between  $\hat{\phi} \sim \kappa$  and  $1/\kappa$ . Note that, for  $\hat{\phi} \lesssim 1$ , the inflaton is a complex scalar field rather than a single real scalar, because the  $\phi_I$  is no longer negligible.

for  $1 \ll \phi_R \ll 1/\sqrt{\kappa}$ . In terms of the canonically normalized field,  $\varphi \equiv \phi_R^2/2$ , we have

$$\mathcal{L} \approx \frac{1}{2}(\partial\varphi)^2 - m^2\varphi, \quad (15)$$

for  $1 \ll \varphi \ll 1/\kappa$ . Thus our model is equivalent to the linear term inflation model.

After inflation ends, the inflaton field will oscillate about the origin. Then the  $\phi_I$  is no longer negligible, and the inflaton is expressed by a complex scalar field. As the amplitude decreases, the  $\kappa$ -term becomes more important, and in the end, the kinetic term arises mainly from the  $\kappa$ -term. The Lagrangian is

$$\begin{aligned} \mathcal{L} &\approx \kappa \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi, \\ &= \partial_\mu \hat{\phi}^\dagger \partial^\mu \hat{\phi} - \frac{m^2}{\kappa} \hat{\phi}^\dagger \hat{\phi}, \end{aligned} \quad (16)$$

for  $\hat{\phi} \equiv \sqrt{\kappa}\phi$  much smaller than the Planck scale. The scalar potential is schematically shown in Fig. 1.

The inflaton dynamics is rather simple. Using the slow-roll approximation, the inflaton

field value is parametrized by the e-folding number as

$$\varphi_N \simeq 11\sqrt{\frac{N_e}{60}}, \quad (17)$$

and the power spectrum of the curvature perturbation is given by

$$\Delta_{\mathcal{R}}^2 \simeq \left( \frac{H}{\dot{\varphi}} \frac{H}{2\pi} \right)^2 \simeq \frac{m^2 \varphi_N^3}{12\pi^2}. \quad (18)$$

Using Eq. (17), the mass scale  $m$  is determined to be

$$m \simeq 3.6 \times 10^{13} \text{ GeV} \left( \frac{N_e}{60} \right)^{-3/4}, \quad (19)$$

in order to explain the WMAP result [1],  $\Delta_{\mathcal{R}}^2 = (2.43 \pm 0.11) \times 10^{-9}$ . The physical mass of the inflaton at the potential minimum is given by

$$m_\phi = \frac{m}{\sqrt{\kappa}} = 4 \times 10^{14} \text{ GeV} \left( \frac{\kappa}{10^{-2}} \right)^{-1/2} \left( \frac{m}{4 \times 10^{13} \text{ GeV}} \right). \quad (20)$$

For the linear inflation to last for more than 60 e-foldings, the symmetry breaking parameter  $\kappa$  must satisfy  $\kappa < 0.1$ . In the extreme case of  $\kappa \sim m^2$ , the inflaton mass can be as heavy as the Planck scale.

Let us discuss what happens if we include higher order terms in the Kähler potential. Suppose that the Kähler potential is given by

$$K = \kappa|\phi|^2 + |X|^2 + \sum_{n=1} \frac{c_n}{n} (\phi^2 - \phi^{\dagger 2})^n, \quad (21)$$

where  $c_n$  is a numerical coefficient of order unity. The inflationary path should be such that the Kähler potential takes a (locally) minimum value, since otherwise the scalar potential will blow up and no inflation occurs. There are generically multiple inflationary paths, and, for a large enough  $\phi_R$ , they are given by

$$\phi_I = \frac{\text{const.}}{\phi_R}. \quad (22)$$

Therefore, the kinetic term still takes a form of  $|\phi|^2|\partial\phi|^2$ , and the resultant scalar potential for a canonically normalized field will be a linear term<sup>5</sup>. This is not surprising, because the form of the kinetic term is determined by the shift symmetry we imposed on a composite

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<sup>5</sup> The kinetic term should have a correct sign during and after inflation, which is realized if the Kähler potential is in one of the local minima with respect to the variation of  $\phi_I$ .

scalar. It is only the symmetry-breaking terms (other than the superpotential (8)) which make the potential deviate from the linear term.

The reheating process in supergravity inflation models have been recently studied in a great detail [8–11]. The inflaton  $\phi$  will get maximally mixed with the  $X$  at the potential minimum [12]. Therefore  $\phi$  can decay into the standard model (SM) particles through couplings of  $X$  with the SM sector. For instance, if we introduce the coupling with Higgs doublets,

$$W = \lambda X H_u H_d, \quad (23)$$

where  $\lambda$  is a numerical coefficient and  $H_{u(d)}$  is the up(down)-type Higgs doublet, the reheating temperature will become

$$T_R \sim 10^{10} \text{ GeV} \left( \frac{\lambda}{10^{-5}} \right) \left( \frac{m_\phi}{10^{14} \text{ GeV}} \right)^{1/2}. \quad (24)$$

Here we have assumed that  $H_u H_d$  has a  $R$ -charge 0 and a negative parity under the  $Z_2$  symmetry, and used the relation  $\lambda \sim m$ . Alternatively, if we allow a symmetry-breaking term  $\delta(\phi + \phi^\dagger) = \delta/\sqrt{\kappa}(\hat{\phi} + \hat{\phi}^\dagger)$  in the Kähler potential, the inflaton decays into the SM particles through the gravitational couplings with the top Yukawa interaction and the  $SU(3)_C$  gauge sector [9, 10]. The reheating temperature will become

$$T_R \sim 5 \times 10^6 \text{ GeV} \left( \frac{\delta/\sqrt{\kappa}}{10^{-3}} \right) \left( \frac{m_\phi}{10^{14} \text{ GeV}} \right)^{3/2}. \quad (25)$$

Note that  $\delta$  violates both the shift and  $Z_2$  symmetries. If the inflaton mass  $m_\phi$  is about  $10^{16} \text{ GeV}$ , the reheating temperature becomes about  $10^{10} \text{ GeV}$ , which is high enough for the thermal leptogenesis to work [13]. However, it is in general difficult to satisfy the constraints from the non-thermal gravitino problem [12] when the inflaton decay is induced by the gravitationally suppressed coupling, unless the gravitino is extremely light  $m_{3/2} \leq O(10) \text{ eV}$  [14] or the gravitino is very heavy *and* the  $R$ -parity is broken. We also note that the inflaton has an approximate  $U(1)$  symmetry at the origin and may naturally acquire an inflaton number asymmetry, which is transferred to the baryon asymmetry in the end. Also, due to the approximate  $U(1)$  symmetry, Q-balls [15] may be formed; however, the charge is relatively small and so it does not affect the reheating process.

It is straightforward to extend the above model to a chaotic inflation model with a different power. For instance, if we consider a shift symmetry  $\phi^n \rightarrow \phi^n + \alpha$  and its breaking  $W = \lambda' X \phi^m$ , the scalar potential for a canonical normalized field  $\varphi$  will be proportional to



$\varphi^{2m/n}$  during inflation, and to  $\varphi^{2m}$  after inflation. Such a shift symmetry on a composite field may be realized in a non-linear sigma model. The general feature of our model is therefore that the potential becomes steeper after inflation. The spectral index and the tensor-to-scalar ratio become  $n_s = 1 - (1 + m/n)/N_e$  and  $r = 8m/(nN_e)$ , where  $N_e$  denotes the e-folding number. The linear term model corresponds to  $n = 2$  and  $m = 1$ .

Due to the running kinetic term, there appears an interesting phenomenon. For instance, consider the case of  $n = m = 2$ . Then, it is an usual quadratic chaotic inflation for  $|\phi| > 1$ , but the potential becomes  $\sim |\phi|^4$  after inflation. Therefore, the evolution of the universe after inflation is different from the usual quadratic chaotic inflaton. We may take  $n = m = 3$ , and then the inflaton energy after inflation decreases more quickly than the radiation or non-relativistic matter, which can lead to the enhancement of the gravity waves or baryon asymmetry [16, 17]. This will have an important impact on the future direct gravity wave search experiments such as advanced LIGO [18], LCGT [19], LISA [20] and DECIGO [21]. Note also that, for  $m \geq 2$ , the inflaton field is massless in the SUSY limit, which can relax the thermal and non-thermal gravitino problem [17].

In principle, we may make use of such a scalar field as a curvaton [22] or ungaussiton [23]. The peculiar form of the scalar potential may make it easier for the field to give a sizable contribution to the total energy.

Lastly let us mention the initial condition of the inflation model. Suppose that the  $\phi$  is fluctuating in the linear potential. If the inflation lasts for large number of e-foldings, the fluctuations will approach a certain distribution, which should be broader than the Bunch-Davies distribution [24] for the quadratic potential. So the linear inflation may take place with a larger probability.

The inflation with a running kinetic term has many implications; the potential becomes flatter, making the inflation to occur easily, and the gravity waves can be enhanced at frequencies within the reach of current and future gravity wave experiments. The future observation [25] of  $n_s$ ,  $r$ , a possibly large non-Gaussianity [7], and direct gravity wave experiments will refute or support these models.

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- [1] E. Komatsu *et al.*, [arXiv:1001.4538 [astro-ph.CO]].
  - [2] A. D. Linde, Phys. Lett. B **129**, 177 (1983).
  - [3] D. H. Lyth, Phys. Rev. Lett. **78**, 1861 (1997).
  - [4] M. Kawasaki, M. Yamaguchi, T. Yanagida, Phys. Rev. Lett. **85**, 3572-3575 (2000).
  - [5] L. McAllister, *et al.*, arXiv:0808.0706 [hep-th].
  - [6] G. 't Hooft, in *Recent developments in gauge theories*, (Plenum Press, Cargese, 1980).
  - [7] S. Hannestad, T. Haugbolle, P. R. Jarnhus and M. S. Sloth, arXiv:0912.3527 [hep-ph].
  - [8] M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. D **74**, 023531 (2006) [arXiv:hep-ph/0605091].
  - [9] M. Endo, M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. **B642**, 518-524 (2006).
  - [10] M. Endo, F. Takahashi, T. T. Yanagida, Phys. Lett. **B658**, 236 (2008); Phys. Rev. **D76**, 083509 (2007).
  - [11] T. Asaka, S. Nakamura, M. Yamaguchi, Phys. Rev. **D74**, 023520 (2006). [hep-ph/0604132].
  - [12] M. Kawasaki, F. Takahashi, T. T. Yanagida, Phys. Lett. **B638**, 8 (2006); Phys. Rev. D **74**, 043519 (2006).
  - [13] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
  - [14] M. Viel, J. Lesgourgues, M. G. Haehnelt, S. Matarrese and A. Riotto, Phys. Rev. D **71**, 063534 (2005).
  - [15] S. R. Coleman, Nucl. Phys. **B262**, 263 (1985).
  - [16] S. Mukohyama, *et al* Phys. Lett. B **679**, 6 (2009).

- [17] K. Nakayama and F. Takahashi, in preparation.
- [18] P. Fritschel, arXiv:gr-qc/0308090.
- [19] K. Kuroda *et al.*, Class. Quant. Grav. **19**, 1237 (2002).
- [20] LISA Pre-Phase A Report, 2nd Ed. (1998): <http://www.srl.caltech.edu/lisa/documents/PrePhaseA.pdf>
- [21] N. Seto, S. Kawamura, T. Nakamura, Phys. Rev. Lett. **87**, 221103 (2001). [astro-ph/0108011].
- [22] K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002); D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002); T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)].
- [23] T. Suyama and F. Takahashi, JCAP **0809**, 007 (2008).
- [24] T. S. Bunch and P. C. W. Davies, Proc. Roy. Soc. Lond. A **360**, 117 (1978).
- [25] [Planck Collaboration], arXiv:astro-ph/0604069.